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VIII. *Astronomical Observations. In two Letters from M. Francis de Zach, Professor of Mathematics, and Member of the Royal Academies of Sciences at Marseilles, Dijon, and Lyons, to Mr. Tiberius Cavallo, F. R. S.*

Read December 23, 1784.

S I R,

Lyons, April 4, 1783.

I SEND you the account of the observations on the eclipse of the moon, which I have made together with the rev. Father LE FEVRE, Astronomer at Lyons, in the Observatory called *au grand Collège*; to which I shall add the observations of the vernal equinox; some observations on Jupiter's satellites, made at Marseilles by M. SAINT JACQUES DE SYLVABELLE; and, lastly, a new solution of a problem that occurs in computing the orbits of comets. If you think that these observations do in any way deserve the notice of the Royal Society, I shall be very glad you would communicate them. In order to ascertain the going of the pendulum clock, I took several corresponding altitudes of the sun, which you will find in the following table. On the day of the eclipse the sky was very serene, nothing could be finer, and it continued so during the observation. I determined to use an achromatic telescope of $3\frac{1}{2}$ feet length, that shews objects in their natural position, because the diluted and uncertain termination of the true shadow of the earth appears more perfectly defined by small than by

large telescopes, which magnify too much, and give too great a transit between the penumbra and the true dark shadow. On that account some celebrated astronomers advise to use for the eclipses of the moon no greater telescopes than of four or five feet length. It was remarked at Paris, that in an eclipse of the moon, observed through a telescope of DOLLOND, the focus of its object lens being 30 inches, and likewise through a telescope of five feet length; the eclipse appeared to begin $4' 7''$ sooner, and to end $4' 7''$ later, through the small than through the long telescope; the like has been remarked by several others, and it has been also observed by myself. As to my observations I am tolerably satisfied with them, as they do not differ materially from those of Father LE FEVRE, though it is known that in eclipses of the moon no greater exactness than that of a minute can be obtained. The moon's spots were carefully observed; for it is known, that the mean of the observations of the moon's spots is sufficient to ascertain the longitude of a place to $4''$ or $5''$ nearly. M. DE LA LANDE comparing the observations of the moon's spots in an eclipse, made the 22d of November, 1760, in Vienna, by the Imperial Astronomer Abbé HELL, with those made at the same time in Paris by M. MESSIER, finds the difference of meridians to be $56' 13''$, which agrees very exactly with that ascertained by other means.

Correspondent altitudes of the Sun taken with a quadrant of three-feet radius, in order to adjust the pendulum clock to apparent time.

	Sun's altit.	Ist obser- vation.	Sun's altit.	IId obser- vation.	Sun's altit.	IIIId obser- vation.	Sun's altit.	IVth obser- vation.	Sun's altit.	Vth obser- vation.
18th March, 1783.	° 25 40	h. ' " 8 25 2	° 26 10	h. ' " 8 28 25	° 26 40	h. ' " 8 31 43	° 27 20	h. ' " 8 36 16	° 28	h. ' " 8 40 53
The sun's upper limb at the horizontal wire, eastern side										
Sun's upper limb at the same altitude, western side		15 12 24		15 9 1		15 5 39		15 1 10		14 56 29
Dividing the sum by 2		23 37 26		23 37 26		23 37 22		23 37 26		23 37 22
Sun's center on the meridian as marked by the clock		11 48 43		11 48 43		11 48 41		11 48 43		11 48 41
Equation of the day		12 8 15		12 8 15		12 8 15		12 8 15		12 8 15
Clock slower than equated solar time		0 19 32		0 19 32		0 19 34		0 19 32		0 19 34

The mean of which I put 19' 32"

19th March, 1783.	Sun's Altit.	1st observa- tion.	Sun's Altit.	11d obser- vation.
The sun's upper limb at the horizontal wire of the first telescope on the eastern side	31 30	h. 9 1 56	33	h. 9 13 9
Sun's upper limb at the same altitude on the western side of the meridian		14 31 47		14 20 33
Dividing the sum by 2		23 33 43		23 33 42
Sun's center on the meridian as marked by the pendulum clock		11 46 51½		11 46 51
Equation of the day		12 7 57		12 7 57
Clock slower than equated solar time		0 21 6		0 21 6

Clock slower than equated solar time 19th March 21 6

18th March 19 32

Retarding of the clock upon 23 h. 58' 8" - 1 34

I observed too the mid-day at the great gnomon of the ob-
servatory, and found at the same time the meridian line erro-
neous by 19'', as you will find in the following tables.

	the 17th Mar.	h. / '' 11 50 50	the 18th Mar.	h. / '' 11 48 56	the 19th Mar.	h. / '' 11 47 3
When the center of the sun's image was on the meridian the time pointed by the clock was						
Equations of those days		12 8 33		12 8 15		12 7 57
Retarding on equated solar time		17 43		19 19	the	20 54
Retarding the 17th				17 43	18th	19 19
Retarding of the clock during those 24 hours				1 36		1 35

I fixed therefore the retarding of the clock 1' 35''.

True

True mid-day concluded by the sun's correspondent altitudes as the clock marked	the 18th Mar.	h. ' "	the 19th Mar.	h. ' "
Equation of the mid-day		11 48 42	11 46 51	
Retarding of the clock at the rate of 1' 35" per 24 hours		- 18	- 18	
		+ 13	+ 11	
True mid-day the pendulum clock should have marked		11 48 37	11 46 44	
Mid-day concluded at the gnomon of the observatory		11 48 56	11 47 3	
Difference, the error of the meridian line or gnomon		19	19	

From thence I concluded,

Mid-day at true solar time	h. ' "	Mid-day at equated solar time	h. ' "
Mid-day the clock should have marked on the 18th	11 59 60	Mid-day the clock should have marked on the 18th	12 8 15
	11 48 37		11 48 37
Retarding upon true solar time	11 23	Retarding upon equated solar time	19 38

Observations of the moon's eclipse the 18th March, 1783.

My observations with an achromatic telescope of 3½ feet length.	Time marked by the clock.	True or ap- parent time.
I M M E R S I O N S.		
The beginning of the eclipse very doubtful.	h. ' "	h. ' "
Shadow touches Grimaldi	7 41 45	7 53 39
Grimaldi all in the shadow	7 42 54	7 54 48
Shadow touches Mare Humorum	7 49 32	8 1 27
———— Copernicus	8 0 21	8 12 16
Copernicus all in the shadow	8 2 29	8 14 24
Tycho touches the shadow	8 6 18	8 18 13
Eudoxus	8 12 19	8 24 15
———— all in the shadow	8 13 43	8 25 39
Mare Serenitatis touches the shadow	8 16 7	8 28 3
———— the shadow in the middle	8 21 31	8 33 27
———— all in the shadow	8 25 54	8 37 50
Proclus touches the shadow	8 32 21	8 44 18
Mare Crisium touches the shadow	8 33 29	8 45 26
———— shadow in the middle	8 35 36	8 47 33
———— all in the shadow	8 36 56	8 48 53
Total immersion	8 38 57	8 50 55
E M E R S I O N S.		
Beginning of the emerſion	10 19 57	10 32 2
Grimaldi emerging	10 23 33	10 35 38
———— all out of the shadow	10 24 9	10 36 14
Mare Humorum emerging	10 29 34	10 41 39
Total emerſion of Mare Humorum	10 35 37	10 47 43
Copernicus all out of the shadow	10 43 6	10 55 12
Mare Serenitatis all emerged	10 57 32	11 9 39
Mare Crisium all emerged	11 15 44	11 27 51
End of the eclipse	11 20 10	11 32 18
Total duration		3 39 0

Father LE FEVRE's observations with a reflector 55 inches focal length, magnifying 300 times.

	Time by the clock.	Apparent time.
I M M E R S I O N S.		
Grimaldi touches the shadow	7 41 43	7 53 37
Kepler touches the shadow	7 52 2	8 3 57
—— all in the shadow	7 53 24	8 5 19
Copernicus touches the shadow	8 0 22	8 12 17
—— all in the shadow	8 2 26	8 14 21
Mare Serenitatis touches the shadow	8 16 7	8 28 3
—— all in the shadow	8 26 2	8 37 58
Mare Crisium touches the shadow	8 33 28	8 45 25
—— all in the shadow	8 36 56	8 48 53
Total immersion	8 38 54	8 50 52
E M E R S I O N S.		
Beginning of the emerfion	10 19 42	10 31 47
Grimaldi emerged	10 23 24	10 35 29
Kepler all out of the shadow	10 35 43	10 47 49
Copernicus all out	10 43 4	10 55 10
Mare Serenitatis all out	10 57 19	11 9 26
—— Crisium all out	11 15 50	11 27 57
End of the eclipse	11 20 22	11 32 30
Total duration		3 39 20

The observation of the vernal equinox was made at the gnomon. The height of this gnomon, taken from the center of the hole by which the beams of the sun come in, is 1878 lines of a French inch; the distance from the bottom of the gnomon to the equinoctial point is 1928; the distance from the upper limb of the sun's image to the equinoctial point was found 16,7; the distance from the under limb 23,4; the diameter of the hole = 6; therefore the distance from the bottom to the upper limb $1928 - 16,7 + 3 = 1914,3$, to the under limb $1928 + 23,4 - 3 = 1948,4$; which gives the time the equinox happened the 20th of March, 5 h. 56' 52" P.M.

Observations of Jupiter's satellites at Marseilles.

1782		Apparent time.	Observation.
April 3	Immersion of the 1st satellite at	2 22 56	good
May 19	Imm. 1st sat.	2 48 12	good
June 7	Imm. IVth sat. was not total, but its light diminished sensibly without ever disappearing; the sky was serene, and Jupiter had six belts very distinctly.		
20	Emergence of the 1st satellite	1 29 46	good
July 5	Em. 1st sat.	11 43 59	good
13	Em. IIId sat.	9 17 28	good
20	Em. IIIId sat.	9 27 13	good
20	Em. IIId sat.	11 51 59	good
21	Em. 1st sat.	10 1 11	good
27	Imm. IIIId sat.	10 40 33	good
Aug. 6	Em. 1st sat.	8 21 20	good
13	Em. 1st sat.	10 18 49	good
14	Em. IIId sat.	8 55 39	good
Sept. 1	Em. IIIId sat.	9 40 44	doubtful
14	Em. 1st sat.	8 6 48	good

IT is known, that the indirect method to calculate the orbits of comets in a conic section, by means of three observations given, is rendered more easy and expeditious if there is a possibility of drawing a graphical figure that represents nearly the orbit under consideration, by means of which the calculation is directed, and the required elements of the comet's path may be rigorously determined. To draw the orbit of a comet that moves in a parabola or ellipsis, the problem is reduced to find the position of the axis and the perihelial distance; this position of the axis will be determined as soon as the angle is known, that the axis forms with another line, whose position is given; this line may be an ordinate to a given point of the curve, or a tangent, or a radius vector, &c. The latter is to

be employed in preference, because the perihelial distance being a constant quantity, the angle of position then becomes the true anomaly of the comet; but as the data of this problem are only geocentric longitudes and latitudes of the comet, deduced from the immediate observations of right ascension and declination, the heliocentric longitudes and latitudes must first be calculated; but as those data are not sufficient, what is not given must be arbitrarily supposed, *viz.* the shortened distances (*distantias curtatas*). This supposition is changed and altered until the calculation will agree with the three observations, then the difference between two longitudes is the angle comprehended between the two shortened distances in the plane of the ecliptic; the whole reduced to the plane of the comet's orbit by means of the heliocentric latitude, gives the difference between the anomalies comprehended by two radius vectors, the problem then is reduced: two radius vectors being given, with the angle comprehended, to find the two true anomalies, the perihelial distance, and the time the comet puts in running its anomalies.

Let therefore $\gamma \in \omega$ represent the ecliptic at an infinite distance; QPR the apparent elliptical or parabolical path of a comet; S the sun's center; P the comet's perihelion; T the place of the earth when the comet was first observed in C; I the earth's place when the comet was observed in K; $ST = d$, $SI = \delta$, the distances from the earth to the sun at the first and second observation known by astronomical tables; let Cm and Kn be two perpendiculars to the plane of the ecliptic, it will be $Sm = u$, $Sn = v$ the two shortened distances.

The observed geocentric longitude of the comet in $T = a = \text{arc } \gamma \omega G$;
the observed geocentric longitude of the comet in $I = \alpha = \text{arc } \gamma \omega H$;
the geocentric longitude of the sun by tables in $T = b = \text{arc } \gamma \omega \triangle A$;
the geocentric longitude of the sun by tables in $I = \beta = \text{arc } \gamma \omega \triangle B$.

Now for the first observation the angle of elongation is $b - a$;
for the angle $ATG = \text{arc } AG = \angle W \simeq A - \angle W G = \text{long. } \odot - \text{long. comet.} = b - a$;

the angle of the annual parallax $SmT = \frac{\sin. (b-a)d}{u} = e$;

the angle of commutation $mST = 180^\circ - e + (b-a) = f$;

from whence the heliocentric longitude of the comet = $b - 180^\circ + f = g$.

The same at the second observation in L.

Angle of elongation = $\beta - \alpha$;

Angle of annual parallax $\epsilon = \frac{\sin. (\beta - \alpha) d}{v}$;

Angle of commutation $\phi = 180^\circ - \epsilon + (\beta - \alpha)$;

heliocentric longitude of the comet in I = $\beta - 180^\circ + \phi = \gamma$;

putting now the heliocentric latitude seen from S = k ;

the geocentric latitude seen from T = l ;

the heliocentric latitude will be $\frac{\sin. f \cdot \text{tang. } l}{\sin. (b-a)} = \text{tang. } k$;

the same with K it will be $\frac{\sin. \phi \cdot \text{tang. } \lambda}{\sin. (\beta - \alpha)} = \text{tang. } \kappa$ heliocentric latitude in K.

Having thus determined the heliocentric latitudes of two observations, the radius vectors will easily be found in the supposition made for the shortened distances, for they are in the same ratio to the radius vectors as the cosine of the heliocentric latitudes are to the radius = 1; therefore the radius vector m of the first observation will be $= \frac{u}{\cos. k}$ and the radius vector of the second observation $\mu = \frac{v}{\cos. \kappa}$.

Taking now the difference between the found heliocentric longitudes, we get the heliocentric motion of the comet upon the ecliptic between two shortened distances, which is to be reduced upon the comet's orbit, this heliocentric motion is therefore $\gamma - g = m$. Now to reduce this motion we have, first,
finus

$\sinus\ totus = 1$ is to $\cosine\ m ::$ as $\cotangent\ k$ is to the tangent of an angle which I put $= n$, and $90^\circ - k \mp n$ will give an angle which I put $= q$. Lastly, the analogy $\cos. n : \cos. q :: \sin. k :$ will give the cosine of an angle ψ , which is the required motion upon the orbit, or the angle comprehended between the two radius vectors m and μ . Let therefore ECPMND be the apparent parabolic path of a comet; S the sun's center; M and N two places of the comet, the angle MSN equal to its motion in longitude, or the comprehended angle ψ ; P the perihelion; it is required to find the two anomalies PM, PN, that is, PSM and PSN, the perihelial distance SP, and the time the comet employed to come from its perihelion P to M and N.

Resolution.

$SM = m$ $SN = \mu$ $MSN = \psi$ $NSB = x$ $MSB = (\psi \pm x)$ $Parameter = p$	<p>In the right-angled triangle SMR and SNV we have</p> $MR = OS = m \sin. (\psi \pm x) \quad NV = QS = \mu \sin. x;$ <p>therefore $OP = \frac{1}{2}p - m \sin. (\psi \pm x)$ and $PQ = \frac{1}{2}p \mp \mu \sin. x$; but by the nature of the parabola we have $SM = AP + PO$ and $SN = AP + PQ$; that is</p> $m = \frac{1}{2}p - m \sin. (\psi \pm x) \quad \mu = \frac{1}{2}p \mp \mu \sin. x$ $m + m \sin. (\psi \pm x) = \frac{1}{2}p \quad \mu \pm \mu \sin. x = \frac{1}{2}p$ $m (1 + \sin. (\psi \pm x)) = \frac{1}{2}p \quad \mu (1 \mp \sin. x) = \frac{1}{2}p$ <p>and $1 + \sin. (\psi \pm x) = \frac{p}{2m} \quad 1 \pm \sin. x = \frac{p}{2\mu}$; by</p>
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putting into a sum $1 + \sin. (\psi \pm x) + 1 \pm \sin. x = \frac{p}{2m} + \frac{p}{2\mu}$; reduction made $2 \pm \sin. x + \sin. (\psi \pm x) = \left(\frac{m + \mu}{2m\mu}\right) p$; but by trigonometrical formulæ we have $\sin. (\psi \pm x) = \sin. \psi \cos. x \pm \sin. x \cos. \psi$. Substituting this expression in its place we obtain, $2 \pm \sin. x + \sin. \psi \cos. x \pm \sin. x \cos. \psi = \left(\frac{m + \mu}{2m\mu}\right) p$. By the same formulæ we have $\cos.^2 x \mp 1 - \sin.^2 x$ and $\cos. x = \sqrt{1 - \sin.^2 x}$.

The angle x defines therefore the position of the axis and the two anomalies required, the perihelial distance being $p = 2\mu \pm 2\mu \sin. x$, it will be known also by the angle x .

In order to find the time the comet employs in running its anomalies, let the perihelial distance just now investigated p be equal to the radius of the earth's orbit, the parabolic area swept by the radius vector will be by the nature of the parabola $\frac{2}{3} PO \times OM + \frac{1}{2} SO \times OM = \frac{4PO \times OM + 3SO \times OM}{6}$. Now

the periphery of the earth's orbit is $7 : 22 :: 2p : \frac{44}{7}p$;

therefore the whole area $\frac{44}{7}p \cdot \frac{1}{2}p = \frac{22}{7}p^2$. It is known that the velocity of a heavenly body moved in a circular path, is to the velocity in a parabolic path in the ratio $\sqrt{2} : 1$. If the parabolic area of the comet is divided by $\sqrt{2}$ it comes out $\frac{4PO \times OM + 3SO \times MO}{6\sqrt{2}}$ equal to an area that the earth describes in

the very same time; put therefore A equal to the time of a fidereal year, we shall recover the analogy; the whole area of the earth's orbit is to the time in which it is described as the parabolic area is to the time consumed in sweeping it; therefore

$$\frac{22}{7}p^2 : A :: \frac{(4PO + 3SO) MO}{6\sqrt{2}} : \frac{7A(4PO + 3SO) MO}{72p^2\sqrt{2}}; \text{ but } OM =$$

$SM \cdot \sin. \text{anom. } PSM$ and $OS = SM \cdot \cos. \text{anom. } PSM$; let the anomaly be $= \delta$, we have $OM = m \sin. \delta$, and $OS = m \cos. \delta$;

therefore $PO = p - m \cos. \delta$. Substituting we obtain $\frac{7A(4p - 4m \cos. \delta + 3m \cos. \delta) m \sin. \delta}{72p^2\sqrt{2}}$ which is $\frac{7A(4p - m \cos. \delta) m \sin. \delta}{72p^2\sqrt{2}}$,

whereby the time is found in parts of a fidereal year.

I am, &c.

SIR,

S I R,

Lyons, May 4, 1783.

LATELY I received from the Observatory at Marfeilles the observation of the tranfit of Mercury, which happened the 12th Nov. 1782. The sky not being very favourable, only the two internal contacts were observed; the first internal contact was observed by M. ST. JACQUES DE SYLVABELLE, at 3 h. 18' 30'' apparent time; the last internal contact by M. ST. JACQUES, at 4 h. 30' 16''; by M. BERNARD, his Adjunctus, at 4 h. 29' 13''. The nearest distances of Mercury's limb to that of the sun in the northern part of its disk were at

h.	'	"	
3	33	14	} 31 } } 34 } } 19 } parts of the micrometer.
3	42	57	
4	22	17	

The apparent diameter of the sun was 2174 parts of this micrometer: I suppose the before-mentioned 2174 parts = $32^{\circ} 26'',9$. I conclude farther, by the observations, the middle of the tranfit at 3 h. 54' 7'',25, whereas I fix, by interpolation, the distances of the limbs at 3 h. 54' 7'',25 = $35'',6$; I have therefore semi-diameter of the sun = $16' 13'',4 - 35'',6 = 15' 37'',8$ + semi-diameter of Mercury = $6'' = 15' 43'',8$ = to the least distance of centers of the sun and Mercury. By M. DE LA LANDE's tables it is $15' 42''$, only a difference of $1'',8$.

M. WALLOT at Paris has observed this tranfit at the Royal Observatory,

	h.	'	"
First external contact	2	56	28
First internal contact	3	2	3
Second - - -	4	17	18
Second external -	4	22	53

I only add an important remark upon the diameter of Mercury, which the astronomers supposed in this transit = $12''$.

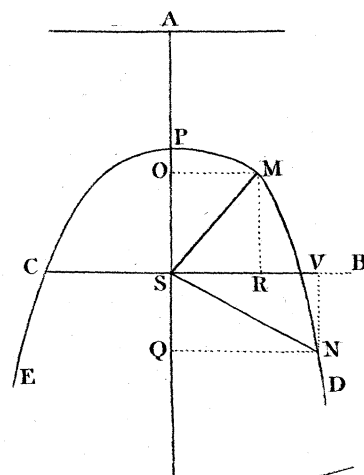
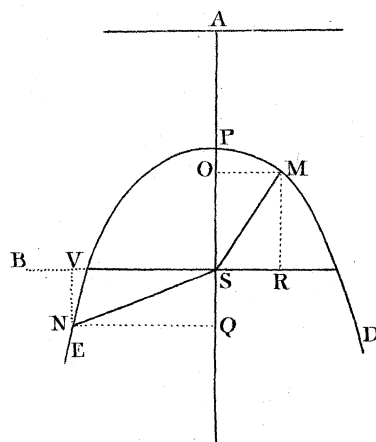
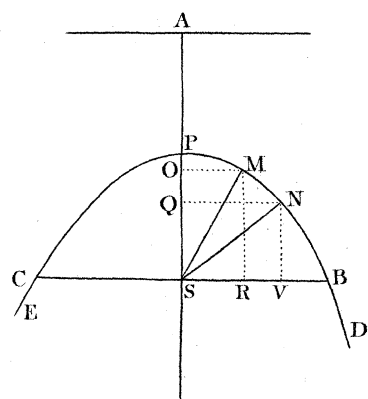
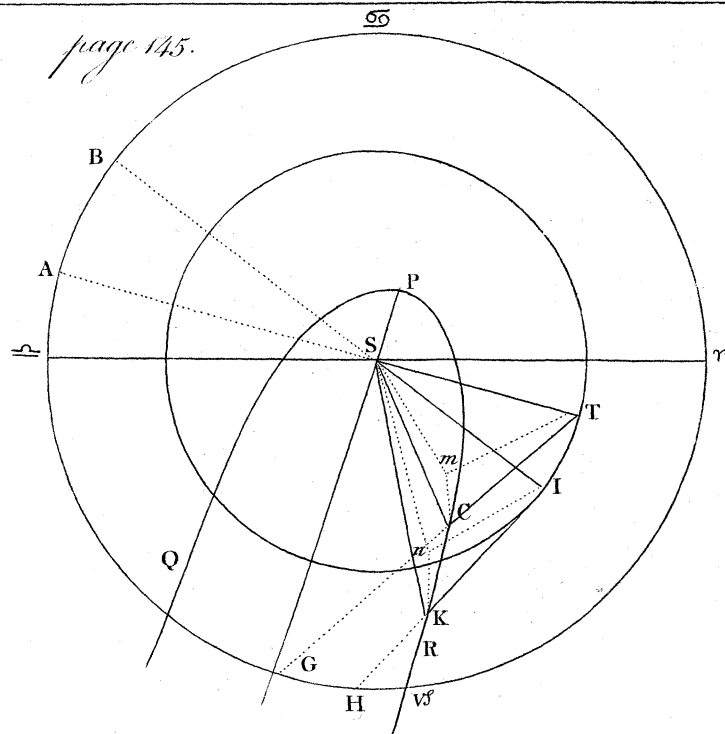
Let ABC represent the sun's disk; in P an external in Q an internal contact; ANC the apparent path of Mercury over the sun.

The femi-diameter of the sun = $972''$, this of Mercury in our supposition = $6''$, MN = $942''$ the least distances of the centers. In the right-angled triangle MNP it is $MP = 972'' + 6'' = 978''$, $MQ = 972'' - 6'' = 966''$; therefore NP will be found = $260''$ and $NQ = 210''$: now $NP - NQ = PQ = 50''$, which converted into time gives $8' 14''$ for the time the diameter of Mercury employed to run over the limb of the sun; but by the observations of M. WALLÖT I find this time constantly in both contacts $5' 35''$; therefore $8' 14'' : 12'' :: 5' 35'' : 8'', 137$, which should be the diameter of Mercury; and indeed M. WALLÖT, by an immediate measure, taken with an excellent wire-micro-meter, finds this apparent diameter not greater than $9''$, which sufficiently shews that this diameter supposed $7''$ in the mean distance is also too great.

I am, &c.



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